Chapter 30

EXPONENTS

An **EXPONENT** is the number of times the **BASE NUMBER** is multiplied by itself.

EXAMPLE: 4^3

4 is the base number. The small, raised number 3 to the right of the base number indicates the number of times the base number is multiplied by itself.

4^3 is read "four to the third power."

Therefore: $4^3 = 4 \times 4 \times 4 = 64$.

COMMON MISTAKE:
The expression 4^3 does NOT mean 4×3.

Things to remember about exponents:

1. Any base without an exponent has an "invisible" exponent of 1.

EXAMPLE: $8 = 8^1$

2. Any base with an exponent 0, equals 1.

EXAMPLE: $6^0 = 1$

3. Be careful when calculating negative numbers with exponents.

EXAMPLE:

$$-3^2 = -(3^2) = -(3 \times 3) = -9 \quad \text{vs.} \quad (-3)^2 = (-3) \times (-3) = 9$$

Always LOOK AT WHAT IS NEXT TO THE EXPONENT:
In the first example, the number 3 is next to the exponent. So, only the 3 is being raised to the second power.

In the second example, the parentheses is next to the exponent, so we raise *everything* inside the parentheses to the second power. The −3 is inside the parentheses and, therefore, −3 is raised to the second power.

Simplifying Expressions with Exponents

You can simplify expressions with more than one exponent by combining the exponents—the only requirement is that the base must be the same. It looks like this:

$$x^a \cdot x^b = x^{a+b}$$
$$x^a \div x^b = x^{a-b}$$

When multiplying powers with the same base, write the base once, and then add the exponents!

EXAMPLE: $5^2 \cdot 5^6 = 5^{2+6} = 5^8$

If you want to check that this works, try the long way:

$$5^2 \cdot 5^6 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^8$$

When dividing powers with the same base, write the base once and subtract the exponents!

EXAMPLE: $7^6 \div 7^2 = 7^{6-2} = 7^4$

If you want to check that this works, try the long way:

$$\frac{7^6}{7^2} = \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7} = 7^4$$

(We can cancel out two of the 7s on top and both on the bottom because anything divided by itself equals 1.)

$$\frac{7^6}{7^2} = \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot \cancel{7} \cdot \cancel{7}}{\cancel{7} \cdot \cancel{7}} = 7^4$$

Let's try it with variables:

EXAMPLE: $x^2 \cdot 2y \cdot x^4$

To simplify, we keep the base (x) and add the exponents $2 + 4$.

$= x^6 \cdot 2y$

← CAN ALSO BE WRITTEN AS $2x^6y$

EXAMPLE: $3a^9 \div 7a^5$

To simplify $a^9 \div a^5$, we keep the base (a) and subtract the exponents $9 - 5$.

$= 3a^4 \div 7$

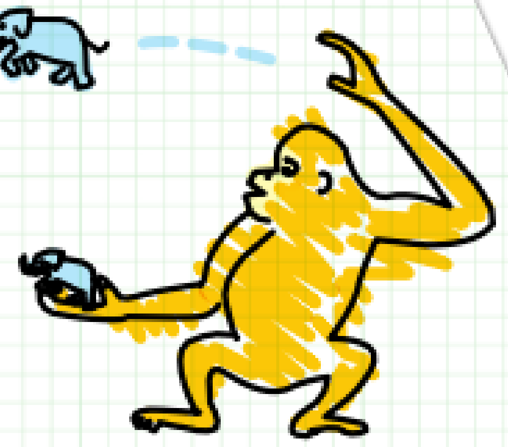
DON'T FORGET THAT YOU CAN ALSO FORMAT THIS QUESTION LIKE A FRACTION IF IT MAKES THE SOLUTION EASIER TO SEE. $\frac{3a^9}{7a^5}$

When there is an exponent inside parentheses and another outside the parentheses, this is called a **POWER OF A POWER**. A power of a power can be simplified by multiplying the exponents. It looks like this:

$$(v^a)^b = v^{a \cdot b}$$

Mnemonic for "Power of a Power: Multiply Exponents":

Powerful **O**rangutans **P**ropelled **M**ultiple **E**lephants.



EXAMPLE: $(4^2)^3 = 4^{2 \cdot 3} = 4^6$

> If you want to check that this works, try the long way:
> $(4^2)^3 = 4^2 \times 4^2 \times 4^2 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

EXAMPLE:

$(3x^7y^4)^2 = 3^{1 \cdot 2} \cdot x^{7 \cdot 2} \cdot y^{4 \cdot 2} = 3^2 \cdot x^{14} \cdot y^8 = 9x^{14}y^8$

(Don't forget: Any base without an exponent has an "invisible" exponent of 1.)

Negative Exponents

What about if you see a **NEGATIVE EXPONENT**? You can easily calculate a negative exponent by using reciprocals.

A negative exponent in the numerator becomes a positive exponent when moved to the denominator. It looks like this:

$$x^{-m} = \frac{1}{x^m}$$

See a negative exponent?

MOVE IT! If it's in the numerator, move it to the denominator and vice versa. Then you can lose the negative sign!

EXAMPLE: $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

And the opposite is true: A negative exponent in the denominator becomes a positive exponent when moved to the numerator. It looks like this:

$$\frac{1}{x^{-m}} = x^m$$

EXAMPLE: $\frac{1}{5^{-2}} = 5^2 = 25$

EXAMPLE: $\frac{x^5 y^{-3}}{x^{-4} y^4}$

Turn y^{-3} into y^3 by moving it to the denominator.

Turn x^{-4} into x^4 by moving it to the numerator.

The new expression is $\frac{x^5 \cdot x^4}{y^3 \cdot y^4}$.

It simplifies to $\frac{x^9}{y^7}$.

CHECK YOUR KNOWLEDGE

Simplify each of the following:

1. 5^3

2. $14m^0$

3. -2^4

4. $x^9 \cdot x^5$

5. $4x^2 \cdot 2y \cdot -3x^5$

6. $\frac{t^9}{t}$

7. $\frac{-15x^4y^2}{5x^3y^2}$

8. $(10^3)^2$

9. $(8m^3n)^3$

10. $\frac{y^5z^{-2}}{y^2z^6}$

ANSWERS

CHECK YOUR ANSWERS

1. 125
2. 14
3. -16
4. x^{14}
5. $-24x^7y$
6. t^8
7. $-3x$
8. 10^6 or 1,000,000
9. $512m^9n^3$
10. $\frac{y^3}{z^8}$