Chapter 28

PROPERTIES

Properties are like a set of math rules that are always true. They often help us solve equations. Here are some important ones:

The **IDENTITY PROPERTY OF ADDITION** looks like this: $a + 0 = a$. It says that if you add zero to any number, that number stays the same.

EXAMPLE: $5 + 0 = 5$

The **IDENTITY PROPERTY OF MULTIPLICATION** looks like this: $a \times 1 = a$. It says that if you multiply any number by 1, that number stays the same.

EXAMPLE: $7 \times 1 = 7$

The **COMMUTATIVE PROPERTY OF ADDITION** looks like this: $a + b = b + a$. It says that when adding two (or more) numbers, you can add them in any order and the answer will be the same.

EXAMPLE: $3 + 11 = 11 + 3$ (Both expressions equal 14.)

The **COMMUTATIVE PROPERTY OF MULTIPLICATION** looks like this: $a \cdot b = b \cdot a$. It says that when multiplying two (or more) numbers, you can multiply them in any order and the answer will be the same.

EXAMPLE: $-5 \cdot 4 = 4 \cdot -5$ (Both expressions equal -20.)

DON'T FORGET: The commutative properties only work with addition and multiplication; they do NOT work with subtraction and division!

When talking about properties, your teacher or textbook may use the term **EQUIVALENT EXPRESSIONS**, which simply means that the math sentences have equal value. For example, $3 + 11 = 11 + 3$. (They are equivalent expressions.)

The **ASSOCIATIVE PROPERTY OF ADDITION** looks like this: $(a + b) + c = a + (b + c)$. It says that when adding three different numbers, you can change the order that you add them by moving the parentheses and the answer will still be the same.

EXAMPLE: $(2 + 5) + 8 = 2 + (5 + 8)$

(Both expressions equal 15.)

The **ASSOCIATIVE PROPERTY OF MULTIPLICATION** looks like this: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. It says that when multiplying 3 different numbers, you can change the order that you multiply them by moving the parentheses and the answer will still be the same.

EXAMPLE: $(2 \cdot 5) \cdot 8 = 2 \cdot (5 \cdot 8)$

(Both expressions equal 80.)

DON'T FORGET: The associative properties only work with addition and multiplication; they do NOT work with subtraction and division!

The **DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION** looks like this: $a(b + c) = ab + ac$. It says that adding two numbers inside parentheses, then multiplying that sum by a number outside the parentheses is equal to first multiplying the number outside the parentheses by each of the numbers inside the parentheses and then adding the two products together.

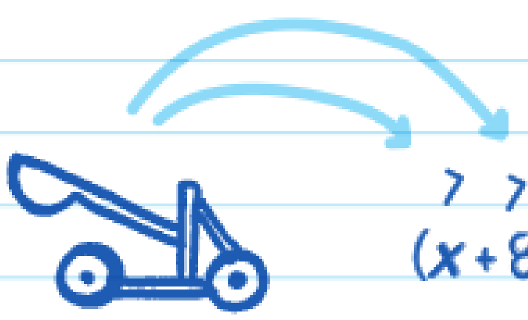
> The **DISTRIBUTIVE PROPERTY** allows us to simplify an expression by taking out the parentheses.

EXAMPLE: $2(4 + 6) = 2 \cdot 4 + 2 \cdot 6$

(You "distribute" the "$2 \cdot$" across the terms inside the parentheses. Both expressions equal 20.)

EXAMPLE: $7(x + 8) =$

> Think about catapulting the number outside the parentheses inside to simplify.



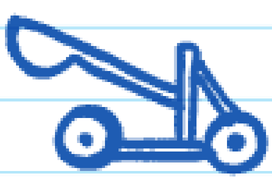
$(x + 8) = 7(x) + 7(8) = 7x + 56$

The **DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER SUBTRACTION** looks like this $a(b - c) = ab - ac$. It says that subtracting two numbers inside parentheses, then multiplying that difference times a number outside the parentheses is equal to first multiplying the number outside the parentheses by each of the numbers inside the parentheses and then subtracting the two products.

EXAMPLE: $9(5 - 3) = 9(5) - 9(3)$

(Both expressions equal 18.)

EXAMPLE: $6(x - 8) =$



$(\overset{6}{x} - \overset{6}{8}) = 6(x) - 6(8) = 6x - 48$

FACTORING is the reverse of the distributive property. Instead of getting rid of parentheses, factoring allows us to include parentheses (because sometimes it's simpler to work with an expression that has parentheses).

EXAMPLE: Factor $15y + 12$.

STEP 1: Ask yourself, "What is the greatest common factor of both terms?" In the above case, the GCF of $15y$ and 12 is 3. ($15y = 3 \cdot 5 \cdot y$ and $12 = 3 \cdot 4$)

STEP 2: Divide all terms by the GCF and put the GCF on the outside of the parentheses.

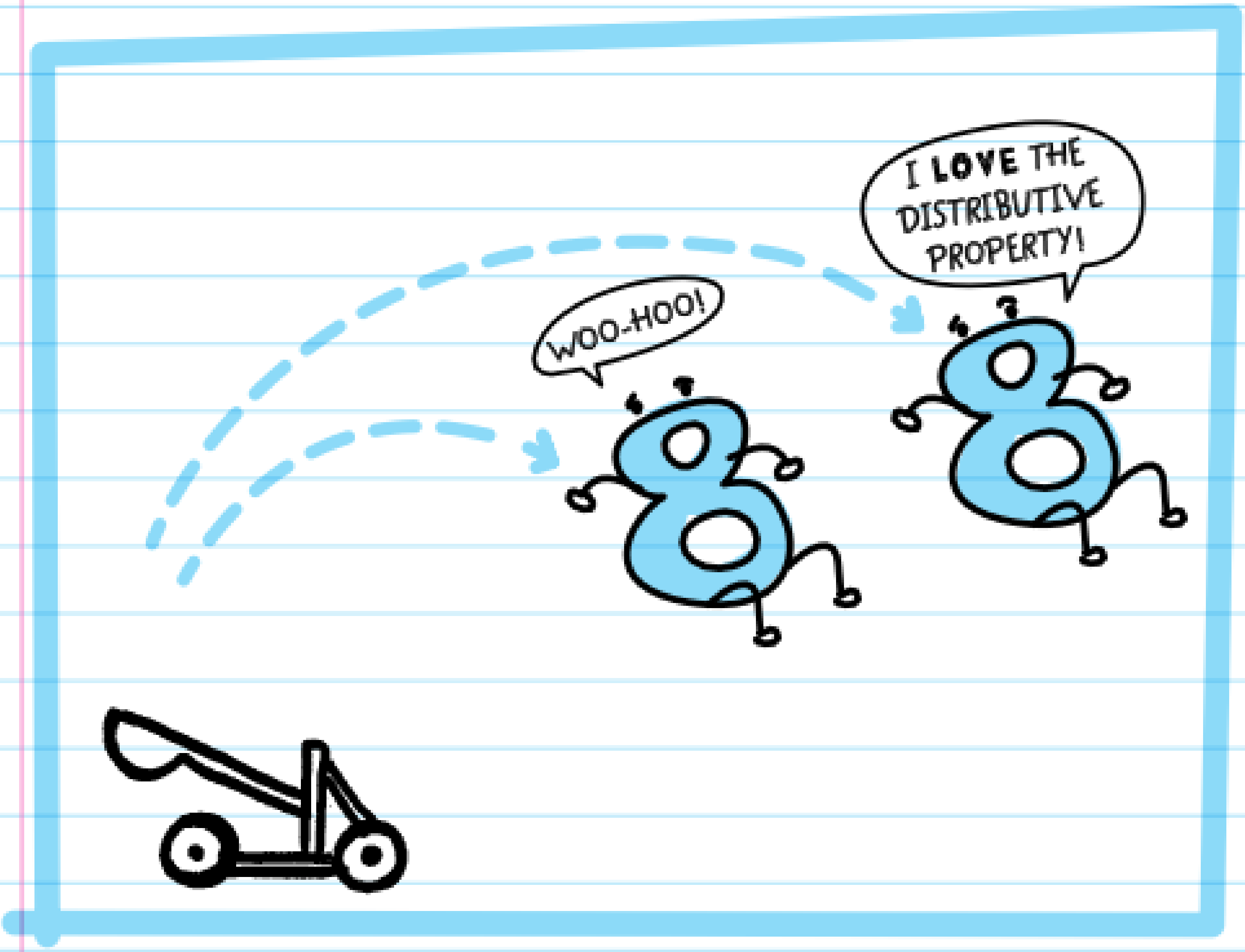
$$15y + 12 = 3(5y + 4)$$

> You can always check your answer by using the **DISTRIBUTIVE PROPERTY**. Your answer should match the expression you started with!

EXAMPLE: Factor $12a + 18$.

The GCF of $12a$ and 18 is 6. So, we divide all terms by 6 and put it outside of the parentheses.

$$12a + 18 = 6(2a + 3)$$


WOO-HOO!
I LOVE THE DISTRIBUTIVE PROPERTY!

CHECK YOUR KNOWLEDGE

In each blank space below, use the property listed to write an equivalent expression.

PROPERTY	EXPRESSION	EQUIVALENT EXPRESSION
Identity Property of Addition	6	
Identity Property of Multiplication	y	
Commutative Property of Addition	$6+14$	
Commutative Property of Multiplication	$8 \cdot m$	
Associative Property of Addition	$(x+4)+9$	
Associative Property of Multiplication	$7 \cdot (r \cdot 11)$	
Distributive Property of Multiplication over Addition	$5(v+22)$	
Distributive Property of Multiplication over Subtraction	$8(7-w)$	
Factor	$18x+6$	
Factor	$14-35z$	

1. Distribute $3(x + 2y - 5)$.

2. Distribute $\frac{1}{2}(4a - 3b - c)$.

3. Factor $6x + 10y + 18$.

4. Factor $3g - 12h - 99j$.

5. Mr. Smith asks Johnny to solve $(12 - 8) - 1$. Johnny says that he can use the Associative Property and rewrite the problem as $12 - (8 - 1)$. Do you agree with Johnny? Why or why not?

ANSWERS

CHECK YOUR ANSWERS

PROPERTY	EXPRESSION	EQUIVALENT EXPRESSION
Identity Property of Addition	6	$6+0$
Identity Property of Multiplication	y	$y \cdot 1$ or $1y$
Commutative Property of Addition	$6+14$	$14+6$
Commutative Property of Multiplication	$8 \cdot m$	$m \cdot 8$
Associative Property of Addition	$(x+4)+9$	$x+(4+9)$
Associative Property of Multiplication	$7 \cdot (r \cdot 11)$	$(7 \cdot r) \cdot 11$
Distributive Property of Multiplication over Addition	$5(v+22)$	$5v+110$
Distributive Property of Multiplication over Subtraction	$8(7-w)$	$56-8w$
Factor	$18x+6$	$6(3x+1)$
Factor	$14-35z$	$7(2-5z)$

1. $3x+6y-15$
2. $2a-\frac{3}{2}b-\frac{1}{2}c$
3. $2(3x+5y+9)$
4. $3(g-4h-33j)$
5. No, Johnny is wrong because the Associative Property does not work with subtraction—the order in which you subtract matters.