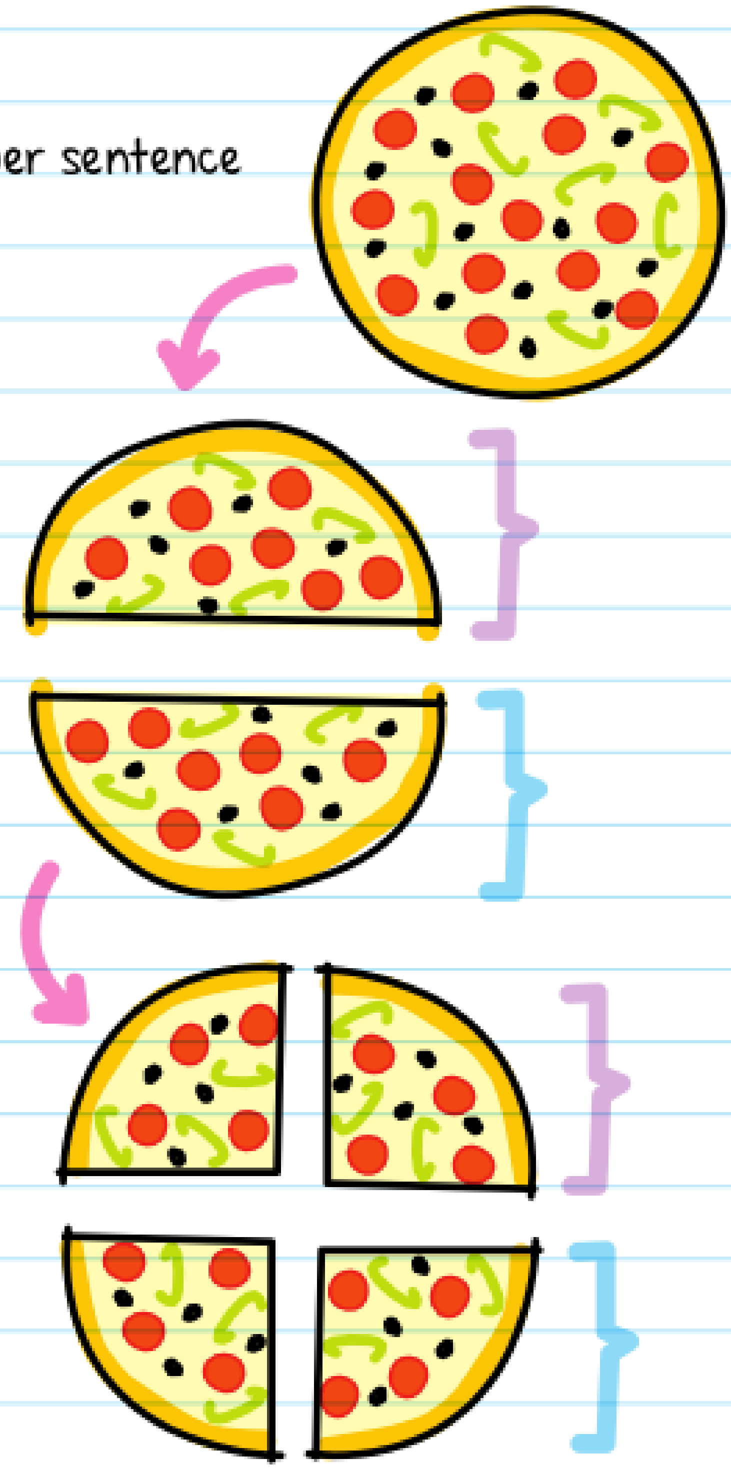
Chapter 17

PROPORTIONS

A **PROPORTION** is a number sentence where two ratios are equal.

For example, someone cuts a pizza into 2 equal pieces and eats 1 piece. The ratio of pieces that person ate to the original pieces of pizza is $\frac{1}{2}$. The number $\frac{1}{2}$ is the same ratio as if that person instead cut the pizza into 4 equal pieces and ate 2 pieces.



$$\frac{1}{2} = \frac{2}{4}$$

You can check if two ratios form a proportion by using cross products. To find cross products, set the two ratios next to each other, then multiply diagonally. If both products are equal to each other, then the two ratios are equal and form a proportion.

$\frac{1}{2} \times \frac{2}{4}$

SOMETIMES, TEACHERS ALSO CALL THIS CROSS MULTIPLICATION.

$1 \times 4 = 4$
$2 \times 2 = 4$

$4 = 4$

The cross products are equal, so $\frac{1}{2} = \frac{2}{4}$.

EXAMPLE: Are $\frac{3}{5}$ and $\frac{9}{15}$ proportional?

$\frac{3}{5} \times \frac{9}{15}$

$3 \times 15 = 45$
$9 \times 5 = 45$

Two ratios that form a proportion are called **EQUIVALENT FRACTIONS.**

$45 = 45$

$\frac{3}{5}$ and $\frac{9}{15}$ ARE proportional—their cross products are equal.

You can also use a proportion to **FIND AN UNKNOWN QUANTITY**. For example, you are making lemonade, and the recipe says to use 5 cups of water for every lemon you squeeze. How many cups of water do you need if you have 6 lemons?

First, set up a ratio: $\frac{5 \text{ cups}}{1 \text{ lemon}}$

Second, set up a ratio for what you are trying to figure out. Because you don't know how many cups are required for 6 lemons, use x for the amount of water.

$$\frac{x \text{ cups}}{6 \text{ lemons}}$$

Third, set up a proportion by setting the ratios equal to each other:

$$\frac{5 \text{ cups}}{1 \text{ lemon}} = \frac{x \text{ cups}}{6 \text{ lemons}}$$

NOTICE THAT THE UNITS ACROSS FROM EACH OTHER MATCH.

Last, use cross products to find the missing number!

$1 \cdot x = 5 \times 6$

$1 \cdot x = 30$ (Divide both sides by 1 so you can get x alone.)

$x = 30$

You need 30 cups for 6 lemons!

EXAMPLE: You drive 150 miles in 3 hours. At this rate, how far would you travel in 7 hours?

$$\frac{150 \text{ miles}}{3 \text{ hours}} = \frac{x \text{ miles}}{7 \text{ hours}}$$

$150 \cdot 7 = 3 \cdot x$

$1{,}050 = 3x$ (Divide both sides by 3 so you can get x alone.)

$350 = x$

You'll travel 350 miles in 7 hours.

Whenever you see "at this rate," set up a proportion!

Sometimes, a proportion stays the same, even in different scenarios. For example, Tim runs $\frac{1}{2}$ a mile, and then he drinks 1 cup of water. If Tim runs 1 mile, he needs 2 cups of water. If Tim runs 1.5 miles, he needs 3 cups of water (and so on). The proportion stays the same, and we multiply by the same number in each scenario (in this case, we multiply by 2). This is known as the **CONSTANT OF PROPORTIONALITY** or the **CONSTANT OF VARIATION** and is closely related to unit rate (or unit price).

EXAMPLE: A recipe requires 6 cups of water for 2 pitchers of fruit punch. The same recipe requires 15 cups of water for 5 pitchers of fruit punch. How many cups of water are required to make 1 pitcher of fruit punch?

We set up a proportion:

$$\frac{6 \text{ cups}}{2 \text{ pitchers}} = \frac{x \text{ cups}}{1 \text{ pitcher}} \quad \text{or} \quad \frac{15 \text{ cups}}{5 \text{ pitchers}} = \frac{x \text{ cups}}{1 \text{ pitcher}}$$

By solving for x in both cases, we find out that the answer is always 3 cups.

We can also see unit rate by using a table. With the data from the table, we can set up a proportion:

EXAMPLE: Daphne often walks laps at the track. The table below describes how much time she walks and how many laps she finishes. How many minutes does Daphne walk per lap?

Total minutes walking	28	42
Total number of laps	4	6

$$\frac{28 \text{ minutes}}{4 \text{ laps}} = \frac{x \text{ minutes}}{1 \text{ lap}} \quad \text{or} \quad \frac{42 \text{ minutes}}{6 \text{ laps}} = \frac{x \text{ minutes}}{1 \text{ lap}}$$

Solving for x, we find out that the answer is 7 minutes.

CHECK YOUR KNOWLEDGE

1. Do the ratios $\frac{3}{4}$ and $\frac{6}{8}$ form a proportion? Show why or why not with cross products.

2. Do the ratios $\frac{4}{9}$ and $\frac{6}{11}$ form a proportion? Show why or why not with cross products.

3. Do the ratios $\frac{4}{5}$ and $\frac{12}{20}$ form a proportion? Show why or why not with cross products.

4. Solve for the unknown: $\frac{3}{15} = \frac{9}{x}$.

5. Solve for the unknown: $\frac{8}{5} = \frac{y}{19}$. Answer in decimal form.

6. Solve for the unknown: $\frac{m}{6.5} = \frac{11}{4}$. Answer in decimal form.

7. In order to make the color pink, a painter mixes 2 cups of white paint with 5 cups of red. If the painter wants to use 4 cups of white paint, how many cups of red paint will she need to make the same color pink?

8. Four cookies cost $7. At this rate, how much will 9 cookies cost?

9. Three bagels cost $2.67. At this rate, how much will 10 bagels cost?

10. It rained 3.75 inches in 15 hours. At this rate, how much will it rain in 35 hours? Answer in decimal form.

ANSWERS

CHECK YOUR ANSWERS

1. Yes, because $\frac{3}{4} \times \frac{6}{8}$
   $3 \times 8 = 24$
   $6 \times 4 = 24$
   $24 = 24$

2. No, because $\frac{4}{9} \times \frac{6}{11}$
   $4 \times 11 = 44$
   $6 \times 9 = 54$
   $44 \neq 54$

3. No, because $\frac{4}{5} \times \frac{12}{20}$
   $4 \times 20 = 80$
   $12 \times 5 = 60$
   $80 \neq 60$

4. $x = 45$

5. $y = 30.4$

6. $m = 17.875$

7. 10 cups

8. $15.75

9. $8.90

10. 8.75 inches